

# HAWKING RADIATION FROM DECOHERENCE

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## **Abstract**

It is argued that the thermal nature of Hawking radiation arises solely due to decoherence. Thereby any information-loss paradox is avoided because for closed systems pure states remain pure. The discussion is performed for a massless scalar field in the background of a Schwarzschild black hole, but the arguments should hold in general. The result is also compared to and contrasted with the situation in inflationary cosmology.

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The process of black-hole evaporation is still not understood on the most fundamental level. One of the key questions concerns the “information-loss paradox” – can a pure quantum state evolve into a mixed state during the evaporation or not [1]? In this Letter I shall argue that Hawking radiation always remains in a pure state and that its mixed appearance emerges through the irreversible process of decoherence [2].

Consider the simplest case of a massless scalar field  $\phi$ . In the situation of an Unruh observer in Minkowski spacetime one usually considers a hypersurface of constant Rindler time which connects the left with the right Rindler wedge. Tracing out in the Minkowski vacuum the modes of the left part leads to a density matrix in the right part corresponding to a canonical ensemble with temperature  $a/2\pi$ , where  $a$  is the acceleration [3]. Alternatively, one can impose the boundary condition  $\phi = 0$  at the origin (corresponding to the presence of a “mirror”). In this case the evolution along the right part of constant Rindler time (called  $t$ ) hypersurfaces is given by the quantum state (denoting the Fourier transform of the scalar field by  $\phi(k)$ ) [4]

$$\Psi \propto \exp \left[ - \int_{-\infty}^{\infty} dk \, k \coth \left( \frac{\pi k}{2a} + ikt \right) |\phi(k)|^2 \right] . \quad (1)$$

(The usual normalisation factor for a Gaussian is being assumed.) The corresponding collapse-situation in the black-hole case was considered in [5] within the CGHS model of two-dimensional gravity. The same quantum state as (1) was found outside the hole, with  $a$  being replaced by the CGHS parameter  $\lambda$ . It is also the case for the eternal hole with the boundary condition  $\phi = 0$  at the bifurcation sphere. I shall assume that the same result holds for a four-dimensional Schwarzschild black hole with  $a$  being replaced by  $\kappa \equiv (4M)^{-1}$ , where  $M$  denotes the mass of the black hole and  $t$  is the Schwarzschild time. The expectation value of the number operator for the mode  $\mathbf{k}$  in this state is then given by the Planck form, independent of  $t$ ,

$$\langle n_{\mathbf{k}} \rangle = \frac{1}{e^{8\pi\omega M} - 1} , \quad (2)$$

where  $\omega = |\mathbf{k}| \equiv k$ .

It was shown in [6] that the vacuum quantum state in a black-hole spacetime is given, for each mode, by a two-mode squeezed vacuum. Such a state also results from an inflationary phase of the early universe. The squeezing parameter is given by

$$\tanh r_k = \exp(-4\pi\omega M) . \quad (3)$$

(Note that the imaginary part of the action for an s-wave outgoing particle in the Hawking radiation is given by  $4\pi\omega(M - \omega/2)$ , being equal to  $-1/2$  times the change of the Bekenstein-Hawking entropy [7].) One recognises that  $r_k \rightarrow 0$  ( $r_k \rightarrow \infty$ ) for

$k \rightarrow \infty$  ( $k \rightarrow 0$ ), i.e. there is no squeezing for small wavelengths, but high squeezing for large wavelengths. For the frequency at the maximum of the Planck spectrum one finds from Wien's law that the squeezing parameter is  $r_k \approx 0.25$  corresponding to an expectation value of particle number  $\langle n_k \rangle = \sinh^2 r_k \approx 0.06$ . This is drastically different from the situation in inflationary cosmology where the distribution is not of Planck form and where squeezing becomes very high for the relevant modes. I assume in the following that space is finite (the black hole being inside a large box), so that one can write  $\Psi = \prod_{\mathbf{k}} \psi_{\mathbf{k}}$  with

$$\psi_{\mathbf{k}} \propto \exp \left[ -k \coth(2\pi k M + ikt) |\phi(k)|^2 \right] . \quad (4)$$

That this does in fact correspond to a squeezed state can be recognised by writing (4) in the form

$$\psi_{\mathbf{k}} \propto \exp \left[ -k \frac{1 + e^{2i\varphi_k} \tanh r_k}{1 - e^{2i\varphi_k} \tanh r_k} |\phi(k)|^2 \right] \equiv \exp \left[ -(\Omega_R + i\Omega_I) |\phi(k)|^2 \right] , \quad (5)$$

with the squeezing parameter  $\tanh r_k = \exp(-4\pi\omega M)$  according to (3) and the squeezing angle  $\varphi_k = -kt$ .

A squeezed state can also be characterised by the contour of the corresponding Wigner function in phase space, which exhibits explicitly both direction and amount of squeezing. For a state of the form (5), the Wigner function for each mode reads (cf. [8, 9, 10] for the analogous situation in cosmology)

$$W(\phi, p) = \frac{1}{\pi} \exp \left[ -\frac{2(p + \Omega_I)^2}{\Omega_R} - 2\Omega_R \phi^2 \right] , \quad (6)$$

where  $\phi$  and  $p$  denote one mode of the field and its momentum, respectively (out of the two real modes in the complex field). The momentum is peaked around its classical value  $p_{cl} = -\Omega_I$  with width  $\Omega_R^{1/2}$ , while the field mode itself is peaked around zero with width  $\Omega_R^{-1/2}$ . Denoting  $4\pi\omega M \equiv x > 0$  one has in particular

$$\Omega_R(t) = \frac{k(1 - e^{-2x})}{1 + e^{-2x} - 2e^{-x} \cos(2kt)} . \quad (7)$$

Evaluating this expression at  $t = 0$  one finds  $\Omega_R(0) > k$ , so that, compared to the ground state where  $\Omega_R = k$ , the state is squeezed in  $\phi$ . Evaluation at  $kt = \pi/2$  leads to  $\Omega_R(\pi/2k) < k$ , so the squeezing is in the  $p$ -direction. One has  $\Omega_R(\pi/2k)/\Omega_R(0) = \tanh^2(2\pi k M)$ , which for the frequency corresponding to the maximum of the Planck spectrum gives  $\approx 0.37$ . The Wigner ellipse rotates around the origin, and the typical timescale of the exchange of squeezing between  $\phi$  and  $p$  is given (again for the frequency of the maximum) by

$$t_k = \frac{\pi}{2k} \approx 14M , \quad (8)$$

which is much smaller than typical observation times. It is for this reason that a coarse-graining with respect to the squeezing angle can be performed. Squeezed states are extremely sensitive to interactions with environmental degrees of freedom [2]. In the present case of quickly rotating squeezing angle this interaction leads to a diagonalisation of the reduced density matrix with respect to the particle-number basis [11]. Thereby the local entropy is maximised, corresponding to the coarse-graining of the Wigner ellipse into a circle. The value of this entropy is given by

$$S_k = (1 + n_k) \ln(1 + n_k) - n_k \ln n_k \xrightarrow{r_k \gg 1} 2r_k. \quad (9)$$

The integration over all modes gives  $S = (2\pi^2/45)T_{BH}^3 V$ , which is just the entropy of the Hawking radiation with temperature  $T_{BH} = (8\pi M)^{-1}$ . In this way, the pure squeezed state becomes indistinguishable from a canonical ensemble with temperature  $T_{BH}$ . The situation is very different in inflationary cosmology where the rotation of the Wigner ellipse is slow, corresponding to the age of the universe for the largest cosmological scales [9]. The entropy is there much smaller than its maximal value  $2r_k$ , which manifests itself in the presence of acoustic peaks in the anisotropy spectrum of the cosmic microwave background [10].

Independent of this practical indistinguishability from a thermal ensemble, the state remains a pure state. In fact, for timescales smaller than  $t_k$  the above coarse-graining is not allowed and the difference to a thermal state could be seen in principle. In the case of a primordial black hole with mass  $M \approx 5 \times 10^{14}$  g, the time is  $t_k \approx 1.7 \times 10^{-23}$  s, which could be of observational significance.

The above consideration refers to hypersurfaces of constant  $t$  which remain outside the horizon. Observations that are performed far outside the horizon should, however, not depend on the location of spacelike hypersurfaces close to the black hole. The above arguments should thus also hold for hypersurfaces which enter the black horizon. One can mimic this situation by considering for simplicity a hypersurface of constant  $t$  in an eternal hole. The Minkowski vacuum  $\Psi_M$  along such a surface connecting regions *III* and *I* in the Kruskal diagram can be written for  $t = 0$  as  $\Psi_M = \prod_{\mathbf{k}} \psi_{\mathbf{k}}$  with [4]

$$\psi_{\mathbf{k}} \propto \exp \left[ -k \coth(4\pi k M) (|\phi_{III}(k)|^2 + |\phi_I(k)|^2) - \frac{k}{\sinh(4\pi k M)} (\phi_{III}^*(k) \phi_I(k) + \phi_I^*(k) \phi_{III}(k)) \right], \quad (10)$$

where  $\phi_I$  and  $\phi_{III}$  denote the modes of the scalar field in the Kruskal regions *I* and *III*, respectively. (Note the occurrence of  $4\pi k M$  instead of  $2\pi k M$  in (4).) As is well known, integrating out the modes  $\phi_{III}$  from (10) leads to a thermal density matrix in *I* with temperature  $T_{BH}$  [3]. This is of course due to the fact that the Minkowski vacuum is an entangled state correlating regions *I* and *III*. It is now easy to show

that the state (10) can be written as a product of two squeezed states. Making a unitary transformation to a new basis,

$$\begin{pmatrix} \phi_I \\ \phi_{III} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \chi_1 - \chi_2 \\ \chi_1 + \chi_2 \end{pmatrix}, \quad (11)$$

(10) becomes

$$\psi_{\mathbf{k}} \propto \exp \left[ -k \coth(2\pi k M) |\chi_1(k)|^2 - k \tanh(2\pi k M) |\chi_2(k)|^2 \right] \equiv \psi_1[\chi_1] \otimes \psi_2[\chi_2]. \quad (12)$$

The state  $\psi_1$  directly corresponds to the squeezed state (4), while  $\psi_2$  follows from this state by the replacement  $\varphi_k \rightarrow \varphi_k + \pi/2$  for the squeezing angle. The arguments above concerning decoherence remain true.

Since a mixed state for a closed system in this line of arguments never occurs, one can expect that unitarity is preserved during the whole black-hole evolution. The above formalism is of course only valid as long as the gravitational background remains fixed, but it is not expected that the inclusion of back reaction changes this scenario. There is thus no “information-loss paradox” in the first place.

The above discussion has not yet addressed the issue of the Bekenstein-Hawking entropy  $S_{BH}$ . One might think, however, that  $S_{BH}$  could also be understood along these lines. For this purpose one would need the inclusion of the correct *gravitational* wave function, at least within the semiclassical approximation (see e.g. [12] for some attempts in this direction). The universal nature of  $S_{BH}$  would then arise due to the decohering influence of environmental degrees of freedom on this wave function. It has even been argued that black holes come into existence only by decoherence [13]. The universality should then hold independent of the existing microscopic degrees of freedom, such as D-branes, at least for black holes that are much heavier than the Planck mass.

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